Practice Exercises

PART A. NO CALCULATOR—DIRECTIONS: Answer these questions *without* using your calculator.

A1. $\lim\limits_{x\to 2}\dfrac{x^2-4}{x^2+4}$ is

(A) 1
(B) 0
(C) $-\dfrac{1}{2}$
(D) ∞

A2. $\lim\limits_{x\to\infty}\dfrac{4-x^2}{x^2-1}$ is

(A) 1
(B) 0
(C) -1
(D) ∞

A3. $\lim\limits_{x\to 3}\dfrac{x-3}{x^2-2x-3}$ is

(A) 0
(B) 1
(C) $\dfrac{1}{4}$
(D) ∞

A4. $\lim\limits_{x\to 0}\dfrac{x}{x}$ is

(A) 1
(B) 0
(C) -1
(D) nonexistent

A5. $\lim\limits_{x\to 2}\dfrac{x^3-8}{x^2-4}$ is

(A) 1
(B) 3
(C) 6
(D) ∞

A6. $\lim\limits_{x\to\infty}\dfrac{4-x^2}{4x^2-x-2}$ is

(A) -2
(B) $-\dfrac{1}{4}$
(C) 1
(D) 2

A7. $\lim\limits_{x\to-\infty}\dfrac{5x^3+27}{20x^2+10x+9}$ is

(A) $-\infty$
(B) -1
(C) 3
(D) ∞

A8. $\lim\limits_{x\to\infty}\dfrac{3x^2+27}{x^3-27}$ is

(A) 0
(B) 1
(C) 3
(D) ∞

A9. $\lim\limits_{x\to\infty}\dfrac{2^{-x}}{2^x}$ is

(A) -1
(B) 1
(C) 0
(D) ∞

A10. $\lim\limits_{x\to-\infty}\dfrac{2^{-x}}{2^x}$ is

(A) -1
(B) 1
(C) 0
(D) ∞

A11. $\lim\limits_{x\to 0}\dfrac{\sin 5x}{x}$

(A) $=\dfrac{1}{5}$
(B) $=1$
(C) $=5$
(D) does not exist

A12. $\lim\limits_{x\to 0}\dfrac{\sin 2x}{3x}$

(A) $=\dfrac{2}{3}$
(B) $=1$
(C) $=\dfrac{3}{2}$
(D) does not exist

A13. The graph of $y = \arctan x$ has

(A) vertical asymptotes at $x = 0$ and $x = \pi$
(B) horizontal asymptotes at $y = \pm\frac{\pi}{2}$
(C) horizontal asymptotes at $y = 0$ and $y = \pi$
(D) vertical asymptotes at $x = \pm\frac{\pi}{2}$

A14. The graph of $y = \frac{x^2 - 9}{3x - 9}$ has

(A) a vertical asymptote at $x = 3$
(B) a horizontal asymptote at $y = \frac{1}{3}$
(C) a removable discontinuity at $x = 3$
(D) an infinite discontinuity at $x = 3$

A15. $\lim\limits_{x\to 0} \frac{\sin x}{x^2 + 3x}$ is

(A) 1
(B) $\frac{1}{3}$
(C) 3
(D) ∞

A16. $\lim\limits_{x\to 0} \sin\frac{1}{x}$ is

(A) ∞
(B) 1
(C) -1
(D) nonexistent

A17. Which statement is true about the curve $y = \frac{2x^2 + 4}{2 + 7x - 4x^2}$?

(A) The line $x = -\frac{1}{4}$ is a vertical asymptote.
(B) The line $x = 1$ is a vertical asymptote.
(C) The line $y = -\frac{1}{4}$ is a horizontal asymptote.
(D) The line $y = 2$ is a horizontal asymptote.

A18. $\lim\limits_{x\to\infty} \frac{2x^2 + 1}{(2 - x)(2 + x)}$ is

(A) -2
(B) 1
(C) 2
(D) nonexistent

A19. $\lim\limits_{x\to 0} \frac{|x|}{x}$ is

(A) 0
(B) 1
(C) -1
(D) nonexistent

A20. $\lim\limits_{x\to\infty} x\sin\frac{1}{x}$ is

Challenge

(A) 0
(B) -1
(C) 1
(D) nonexistent

A21. $\lim\limits_{x\to\pi} \frac{\sin(\pi - x)}{(\pi - x)}$ is

(A) 1
(B) 0
(C) π
(D) nonexistent

A22. Let $f(x) = \begin{cases} \frac{x^2 - 1}{x - 1} & \text{if } x \neq 1 \\ 4 & \text{if } x = 1 \end{cases}$

Which of the following statements is (are) true?

I. $\lim\limits_{x\to 1} f(x)$ exists
II. $f(1)$ exists
III. f is continuous at $x = 1$

(A) I only
(B) II only
(C) I and II only
(D) I, II, and III

A23. If $\begin{cases} f(x) = \frac{x^2 - x}{2x} & \text{for } x \neq 0 \\ f(0) = k \end{cases}$

and if f is continuous at $x = 0$, then $k =$

(A) -1
(B) $-\frac{1}{2}$
(C) $\frac{1}{2}$
(D) 1

A24. Suppose $\begin{cases} f(x) = \frac{3x(x - 1)}{x^2 - 3x + 2} & \text{for } x \neq 1, 2 \\ f(2) = -3 \\ f(2) = 4 \end{cases}$

Then $f(x)$ is continuous

(A) except at $x = 1$
(B) except at $x = 2$
(C) except at $x = 1$ or 2
(D) at each real number

A25. The graph of $f(x) = \dfrac{4}{x^2 - 1}$ has

(A) one vertical asymptote, at $x = 1$
(B) the y-axis as its vertical asymptote
(C) the x-axis as its horizontal asymptote and $x = \pm 1$ as its vertical asymptotes
(D) two vertical asymptotes, at $x = \pm 1$, but no horizontal asymptote

A26. The graph of $y = \dfrac{2x^2 + 2x + 3}{4x^2 - 4x}$ has

(A) a horizontal asymptote at $y = \frac{1}{2}$ but no vertical asymptote
(B) no horizontal asymptote but two vertical asymptotes, at $x = 0$ and $x = 1$
(C) a horizontal asymptote at $y = \frac{1}{2}$ and two vertical asymptotes, at $x = 0$ and $x = 1$
(D) a horizontal asymptote at $y = \frac{1}{2}$ and two vertical asymptotes, at $x = \pm 1$

A27. Let $f(x) = \begin{cases} \dfrac{x^2 + x}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$

Which of the following statements is (are) true?

I. $f(0)$ exists
II. $\lim\limits_{x \to 0} f(x)$ exists
III. f is continuous at $x = 0$

(A) I only
(B) II only
(C) I and II only
(D) I, II, and III

PART B. CALCULATOR ACTIVE—DIRECTIONS: Some of the following questions require the use of a graphing calculator.

B1. If $[x]$ denotes the greatest integer not greater than x, then $\lim\limits_{x \to 1/2} [x]$ is

(A) $\frac{1}{2}$
(B) 1
(C) 0
(D) nonexistent

B2. If $[x]$ denotes the greatest integer not greater than x, then $\lim\limits_{x \to -2} [x]$ is

(A) -3
(B) -2
(C) -1
(D) nonexistent

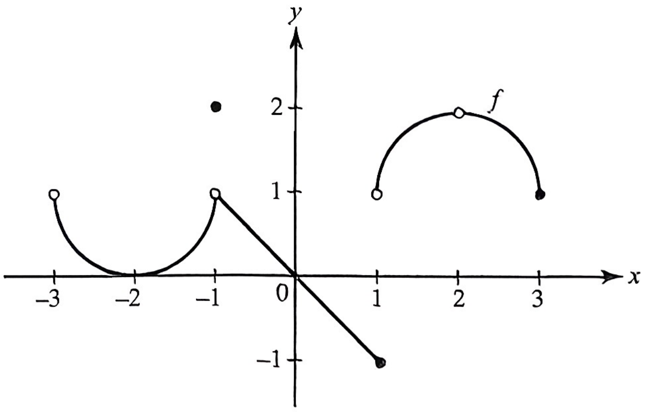
B3. $\lim\limits_{x \to \infty} \sin x$

(A) is -1
(B) is infinity
(C) is zero
(D) does not exist

B4. The function $f(x) = \begin{cases} \dfrac{x^2}{x} & (x \neq 0) \\ 0 & (x = 0) \end{cases}$

(A) is continuous everywhere
(B) is continuous except at $x = 0$
(C) has a removable discontinuity at $x = 0$
(D) has an infinite discontinuity at $x = 0$

Questions B13–B15 are based on the function f shown in the graph.



B13. For what value(s) of a is it true that $\lim_{x \to a} f(x)$ exists and $f(a)$ exists, but $\lim_{x \to a} f(x) \neq f(a)$? It is possible that $a =$

(A) -1 only
(B) 2 only
(C) -1 or 1 only
(D) -1 or 2 only

B14. $\lim_{x \to a} f(x)$ does not exist for $a =$

(A) -1 only
(B) 1 only
(C) 2 only
(D) 1 and 2 only

B15. Which of the following statements about limits at $x = 1$ is (are) true?

I. $\lim_{x \to 1^-} f(x)$ exists
II. $\lim_{x \to 1^+} f(x)$ exists
III. $\lim_{x \to 1} f(x)$ exists

(A) I only
(B) II only
(C) I and II only
(D) I, II, and III