

Limit Exercises

Wednesday, February 12, 2025

1

EXAMPLE 7 The graph of a function g is shown in Figure 10. Use it to state the values (if they exist) of the following:

- (a) $\lim_{x \rightarrow 2^-} g(x)$ (b) $\lim_{x \rightarrow 2^+} g(x)$ (c) $\lim_{x \rightarrow 2} g(x)$
(d) $\lim_{x \rightarrow 5^-} g(x)$ (e) $\lim_{x \rightarrow 5^+} g(x)$ (f) $\lim_{x \rightarrow 5} g(x)$

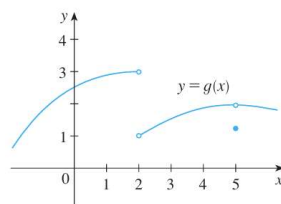


FIGURE 10

2

Find $\lim_{x \rightarrow 3^+} \frac{2x}{x-3}$ and $\lim_{x \rightarrow 3^-} \frac{2x}{x-3}$.

3

11–12 Sketch the graph of the function and use it to determine the values of a for which $\lim_{x \rightarrow a} f(x)$ exists.

$$11. f(x) = \begin{cases} 1 + x & \text{if } x < -1 \\ x^2 & \text{if } -1 \leq x < 1 \\ 2 - x & \text{if } x \geq 1 \end{cases}$$

$$12. f(x) = \begin{cases} 1 + \sin x & \text{if } x < 0 \\ \cos x & \text{if } 0 \leq x \leq \pi \\ \sin x & \text{if } x > \pi \end{cases}$$

4

Find $\lim_{t \rightarrow 0} \frac{\sqrt{t^2 + 9} - 3}{t^2}$.

5

Prove that $\lim_{x \rightarrow 0} \frac{|x|}{x}$ does not exist.

6

Evaluate the limit, if it exists.

$$\lim_{x \rightarrow 5} \frac{x^2 - 6x + 5}{x - 5}$$

$$\lim_{t \rightarrow -3} \frac{t^2 - 9}{2t^2 + 7t + 3}$$

$$\lim_{x \rightarrow -2} \frac{x + 2}{x^3 + 8}$$

7

Use the Squeeze Theorem to show that

$$\lim_{x \rightarrow 0} (x^2 \cos 20\pi x) = 0$$

8

If $4x - 9 \leq f(x) \leq x^2 - 4x + 7$ for $x \geq 0$, find $\lim_{x \rightarrow 4} f(x)$.

9

On what intervals is each function continuous?

(a) $f(x) = x^{100} - 2x^{37} + 75$

(b) $g(x) = \frac{x^2 + 2x + 17}{x^2 - 1}$

(c) $h(x) = \sqrt{x} + \frac{x+1}{x-1} - \frac{x+1}{x^2+1}$